Math 0350 Practice Final

1. $3x - 2y = 6$ Eliminate y

$2x + 4y = 20$

$2(3x - 2y) = 2 \cdot 6 \Rightarrow$

$$6x \boxed{-4y} = 12 \quad (1)$$
$$2x \boxed{+4y} = 20 \quad (2)$$

$$\frac{8x}{8} = \frac{32}{8}$$

$$\underline{\underline{x = 4}}$$

$2(4) + 4y = 20 \quad (2)$

$8 + 4y = 20$

$4y = 12$

$y = 3$

$(4, 3)$

2) $7x - y = 24$

$\rightarrow x = 2y + 9$

$7(2y + 9) - y = 24$

$\underline{14y} + 63 \underline{-y} = 24$

$13y + 63 = 24$

$\quad -63 \quad -63$

$$\frac{13y}{13} = \frac{-39}{13}$$

$$\boxed{y = -3}$$

$x = 2(-3) + 9$

$-6 + 9$

$\boxed{x = 3}$

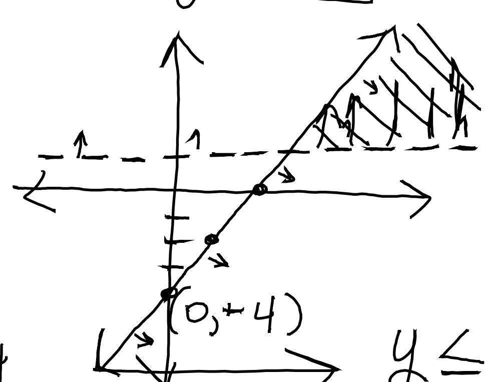
$(3, -3)$

3) $2x - y \geq 4$

$y \geq 1$

$\rightarrow -y \geq -2x + 4$

$$\frac{-y}{-1} \leq \frac{-2x}{-1} + \frac{4}{-1}$$



$y = 1$

$y \leq 2x - 4$

4a) $y = kx \Rightarrow y = 13x$

$26 = k \cdot 2$

$13 = k$

b) $y = \cancel{k}\,\frac{k}{x}$ $\quad 5 = \cancel{4}\,\frac{k}{4}$ $\quad k = 20$

$y = \frac{20}{x}$

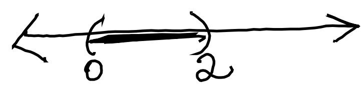
5) $|x-1| + 4 < 5$

$\quad\quad -4 \quad -4$

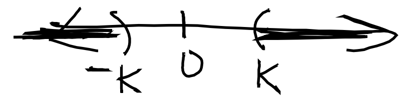
$|x-1| < 1 \Rightarrow |\text{anything}| < \text{pos number}$

$-1 < x-1 < 1 \qquad -a < \text{anything} < a$

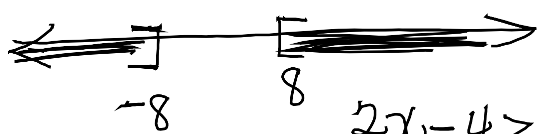
$0 < x < 2 \qquad (0, 2)$



6) $|x| > k \Rightarrow x > k$ or $x < -k$



$|2x-4| \geq 8$



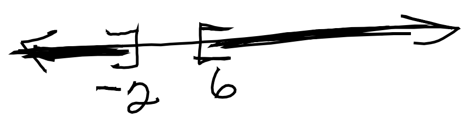
$2x - 4 \geq 8$

$2x - 4 \leq -8$ OR $2x \geq 12$

$\quad +4 \quad +4 \qquad x \geq 6$

$2x \leq -4$

$x \leq -2$



$(-\infty, -2] \cup [6, \infty)$

7) $|3x-1| - 5 = 3$

$\quad\quad +5 \quad +5$

$|3x-1| = 8$

$\rightarrow 3x-1 = 8$ or $3x-1 = -8$

$3x - 1 = 8$

$3x = 9$

$x = 3$

$3x - 1 = -8$

$3x = -7$

$x = -7/3$

(8) $m = 2 \quad p(\overset{x_1}{1}, \overset{y_1}{3})$

$y = mx + b$ or $y - y_1 = m(x - x_1)$

$y - 3 = 2(x - 1)$

$y - 3 = 2x - 2$

$y = 2x + 1$

$f(x) = 2x + 1$

(9) $(0,2)$ $\perp$ to $\frac{1}{3}x + y = 5 \Rightarrow y = -1/3 x + 5$

$m = -1/3$

$m_{\perp} = 3$

$y = mx + b$

$y = 3x + 2$ <u>OR</u> $f(x) = 3x + 2$

(10) $$\left(\frac{-x^3}{27y^{-6}}\right)^{2/3} = \frac{(-1)^{2/3}(x^3)^{2/3}}{(27)^{2/3}(y^{-6})^{2/3}} = \frac{1x^2}{9y^{-4}}$$

$$= \frac{x^2 y^4}{9}$$

(11) $64^{-1/2} = \left(64^{1/2}\right)^{-1} = (8)^{-1} = \frac{1}{8}$

(12) $\sqrt{49x^6y^{16}z^4} = (49)^{1/2}(x^6)^{1/2}(y^{16})^{1/2}(z^4)^{1/2}$

$= 7x^3y^8z^2$

(13) $\sqrt[3]{\frac{48x^8y^{14}}{2x^2y^2}} = \sqrt[3]{24x^6y^{12}} = \sqrt[3]{8 \cdot 3\, x^6 y^{12}}$

$= \underline{\sqrt[3]{8}}\ \underline{\sqrt[3]{x^6}}\ \underline{\sqrt[3]{y^{12}}}\ \sqrt[3]{3}$

$\boxed{2x^2y^4\sqrt[3]{3}}$

(14) $\sqrt{27} - \sqrt{48}$

$\sqrt{9 \cdot 3} - \sqrt{16 \cdot 3}$

$\sqrt{9}\sqrt{3} - \sqrt{16}\sqrt{3}$

$3\sqrt{3} - 4\sqrt{3} = -\sqrt{3}$

(15) $\sqrt[3]{\underline{32x^4}} + 6x\sqrt[3]{4x}$

$\sqrt[3]{8 \cdot 4\, x^3 x^1}$

$\sqrt[3]{8}\ \sqrt[3]{4}\ \sqrt[3]{x^3}\ \sqrt[3]{x^1}$

$\underline{2x}\sqrt[3]{4x} + \underline{6x}\sqrt[3]{4x}$

$8x\sqrt[3]{4x}$

(16) $(4\sqrt{3} - 3\sqrt{2})(5\sqrt{3} + 3\sqrt{2})$

F $\quad 20\sqrt{9} \rightarrow 20 \cdot 3 = 60$

O $\quad 12\sqrt{6}$

I $\quad -15\sqrt{6} \quad\quad -3\sqrt{6}$

L $\quad -9\sqrt{4} \rightarrow -9 \cdot 2 \quad -18$

$42 - 3\sqrt{6}$

(17) $\dfrac{\sqrt{5}-1}{\sqrt{5}+1} \cdot \dfrac{\sqrt{5}-1}{\sqrt{5}-1} = \dfrac{\sqrt{25} - \sqrt{5} - \sqrt{5} + 1}{\sqrt{25} - \cancel{\sqrt{5}} + \cancel{\sqrt{5}} - 1}$

$= \dfrac{5 - 2\sqrt{5} + 1}{5 - 1} = \dfrac{6 - 2\sqrt{5}}{4}$

$= \dfrac{\cancel{2}^{1}(3 - \sqrt{5})}{\cancel{4}\,2} = \dfrac{3 - \sqrt{5}}{2}$

(18) $\dfrac{\sqrt{16}}{\sqrt{3}} = \dfrac{4}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$

(19) $\sqrt{2x+1} + 5 = 8 \Rightarrow \sqrt{2x+1} = 3$

SQ BOTH SIDES CK

19 continued $\sqrt{2x+1} = 3$

$$2x+1 = 9$$

$$2x = 8$$

$$x = 4$$ ✓

ck $\sqrt{2x+1} + 5 = 8$

$$\sqrt{8+1} + 5 \stackrel{?}{=} 8$$

$$3+5=8$$

(20) $$\left(\sqrt[3]{4-2x}\right)^3 = (2)^3$$

$$4-2x = 8$$

$$-2x = 4$$

$$x = -2$$

(21) $$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

x_1, y_1
$(-5, 8)$

$(1, 16)$
x_2, y_2

$$= \sqrt{(1-(-5))^2 + (16-8)^2}$$

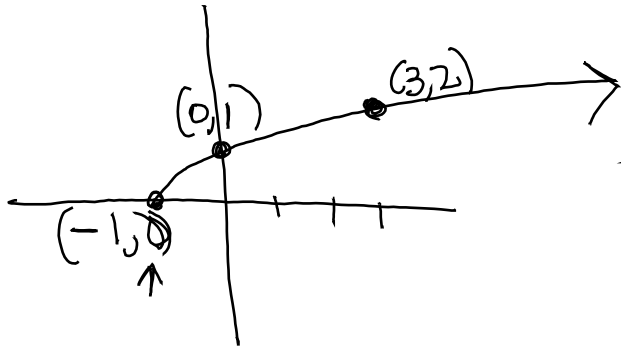
$$\sqrt{36+64} = \sqrt{100} = 10 \text{ units}$$

(22) $f(x) = \sqrt{x+1}$

$x+1 \geq 0 \leftarrow$

$x \geq -1$ $\quad$ D: $[-1, \infty)$

R: $[0, \infty)$



x	$\sqrt{x+1}$
-1	$\sqrt{-1+1} = \sqrt{0} = 0$
0	$\sqrt{0+1} = \sqrt{1} = 1$
3	$\sqrt{3+1} = \sqrt{4} = 2$

(23) $x^2 - 4x - 1 = 0$

$x^2 - 4x + 4 = 1 + 4$ $\qquad \left(\frac{-4}{2}\right)^2$

$(x-2)^2 = 5$

Use sq root prop.

$x - 2 = \pm\sqrt{5}$

$x = 2 \pm \sqrt{5}$ $\qquad 2+\sqrt{5}$, $\; 2-\sqrt{5}$

(24) $ax^2 + bx + c = 0$ $\qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x^2 - 6x = -6$

$x^2 - 6x + 6 = 0$ $\qquad x = \frac{-(-6) \pm \sqrt{36 - 4(1)(6)}}{2}$

$a = 1 \quad b = -6 \quad c = 6$ $\qquad x = \frac{6 \pm \sqrt{36-24}}{2}$

$$X = \frac{6 \pm \sqrt{12}}{2} = \frac{6 \pm 2\sqrt{3}}{2} = \frac{\cancel{2}^{1}(3 \pm \sqrt{3})}{\cancel{2}}$$

$$x = 3 + \sqrt{3} \quad \text{or } 3 - \sqrt{3}$$

25 $(6 - \sqrt{-36}) - (4 + \sqrt{-49})$

$(6 - 6i) - (4 + 7i)$

$6 - 6i - 4 - 7i = 2 - 13i$

26 $(5 - 3i)(4 - 2i)$

F: 20
O: $-10i$
I: $-12i$
L: $6i^2$

$6(-1)$

$$\Longrightarrow \begin{array}{r} 20 \\ -10i \\ -12i \\ -6 \\ \hline \end{array} \quad 14 - 22i$$

27 $\frac{2 - i}{3 + i} \cdot \frac{3 - i}{3 - i} = \frac{6 - 2i - 3i + \overset{-1}{i^2}}{9 - i^2} = \frac{5 - 5i}{10}$

$9 - (-1)$

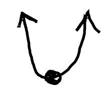
10

$\frac{5}{10} - \frac{5}{10}i = \frac{1}{2} - \frac{1}{2}i$

(28) a $f(x) = x^2 - 2x - 3$ $a = 1 > 0$

x @ vertex $= -\frac{b}{2a}$ $b = -2$

$c = -3$



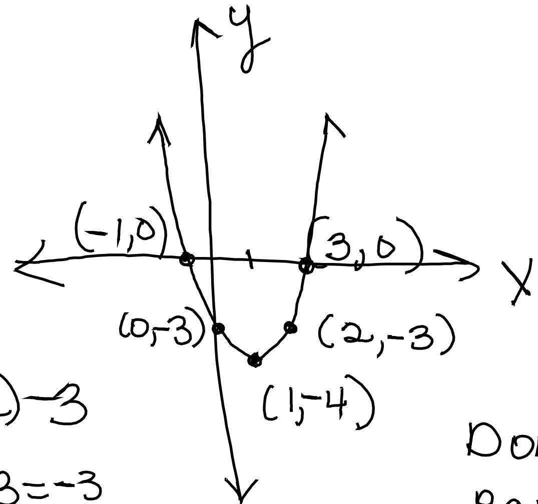
$$x = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$$

$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$$f(1) = 1^2 - 2(1) - 3 = 1 - 2 - 3 = -4$$

vertex $(1, -4)$

x	y
0	−3
+1	−4
2	



$f(2) = 2^2 - 2(2) - 3$

$4 - 4 - 3 = -3$

Domain $(-\infty, \infty)$

Range $[-4, \infty)$

$$0 = x^2 - 2x - 3$$

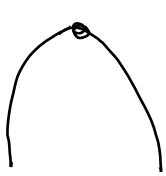
$$0 = (x-3)(x+1)$$

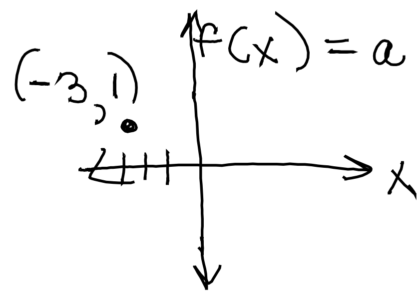
$x - 3 = 0$ $x + 1 = 0$

$x = 3$ $x = -1$

b) $f(x) = -(x+3)^2 + 1$ vertex: $(h, k) = (-3, 1)$

$f(x) = a(x-h)^2 + k$





x	y
−4	0
−3	1
−2	

x	y
-4	0
-3	1
-2	0

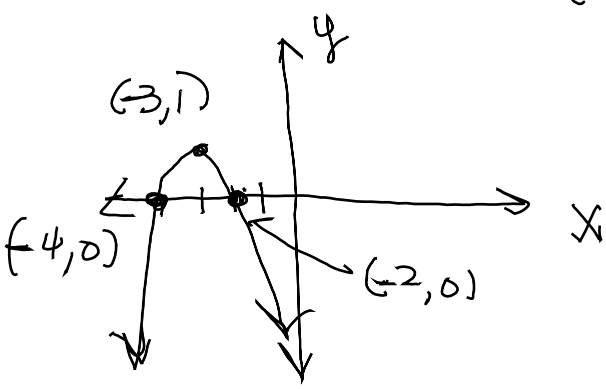
$f(x) = -(x+3)^2 + 1$

$f(-4) = -(-4+3)^2 + 1$

$-(-1)^2 + 1 = -1+1 = 0$

$f(-2) = -(-2+3)^2 + 1$

$-(1)^2 + 1 = -1+1$



Domain $(-\infty, \infty)$

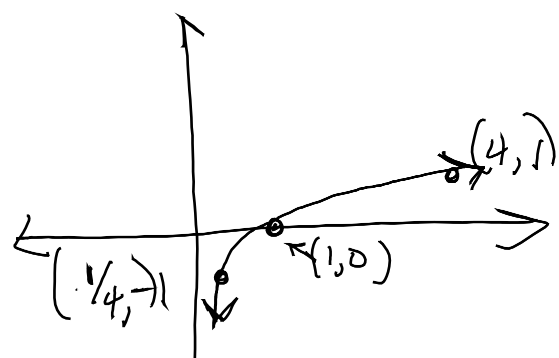
Range $(-\infty, 1]$

29) $f(x) = \log_4 x \longleftrightarrow y = \log_4 x$

$4^y = x$

y	x
-1	$4^{-1} = 1/4$
0	$4^0 = 1$
1	$4^1 = 4$

x	y
$\frac{1}{4}$	-1
1	0
4	1

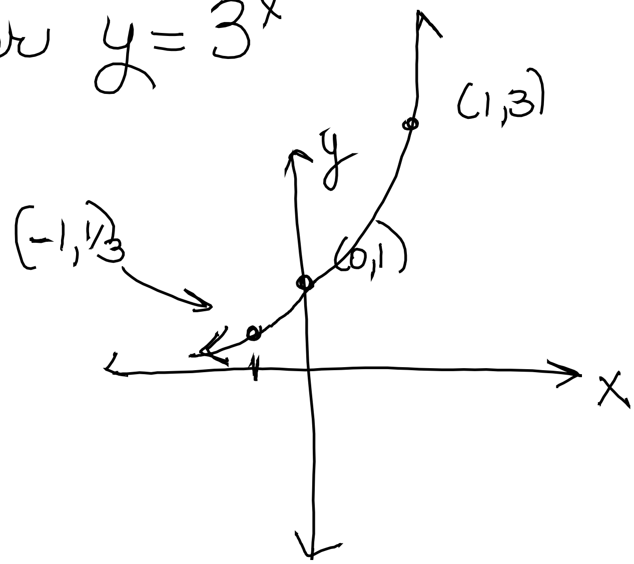


Domain $(0, \infty)$

Range $(-\infty, +\infty)$

(30) $f(x)=3^x$ or $y=3^x$

x	y
-1	$3^{-1}=\frac{1}{3}$
0	$3^0=1$
1	$3^1=3$



Domain $(-\infty,\infty)$
Range $(0,\infty)$

(31) $\log_x 81=4$

$x^4=81=3^4$

$x=3$

(32) $\log_{10} 100=x$

$10^x=100=10^2$

$x=2$

$\log_{10} 10^2=2$

$\log_b b^x=x$

(33) $\log_2 x=4$

$2^4=x$

$16=x$

(34) $\log a^6b^2c = \log a^6+\log b^2+\log c$

$= \boxed{6\log a+2\log b+\log c}$

(35) $\frac{1}{3}\ln a+7\ln b-\ln c = \ln a^{1/3}+\ln b^7-\ln c = \ln(a^{1/3}b^7)-\ln(c)$

$\ln\left(\frac{a^{1/3}b^7}{c}\right) = \ln\left(\frac{\sqrt[3]{a}\;b^7}{c}\right)$

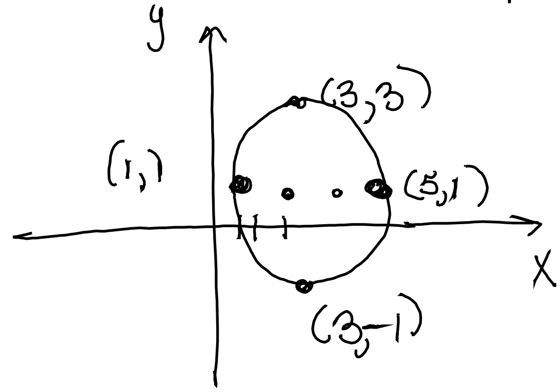
36) $\log_b X = \dfrac{\log X}{\log b} = \dfrac{\ln X}{\ln b}$

$\log_4 5 = \dfrac{\log 5}{\log 4} \approx 1.1610$

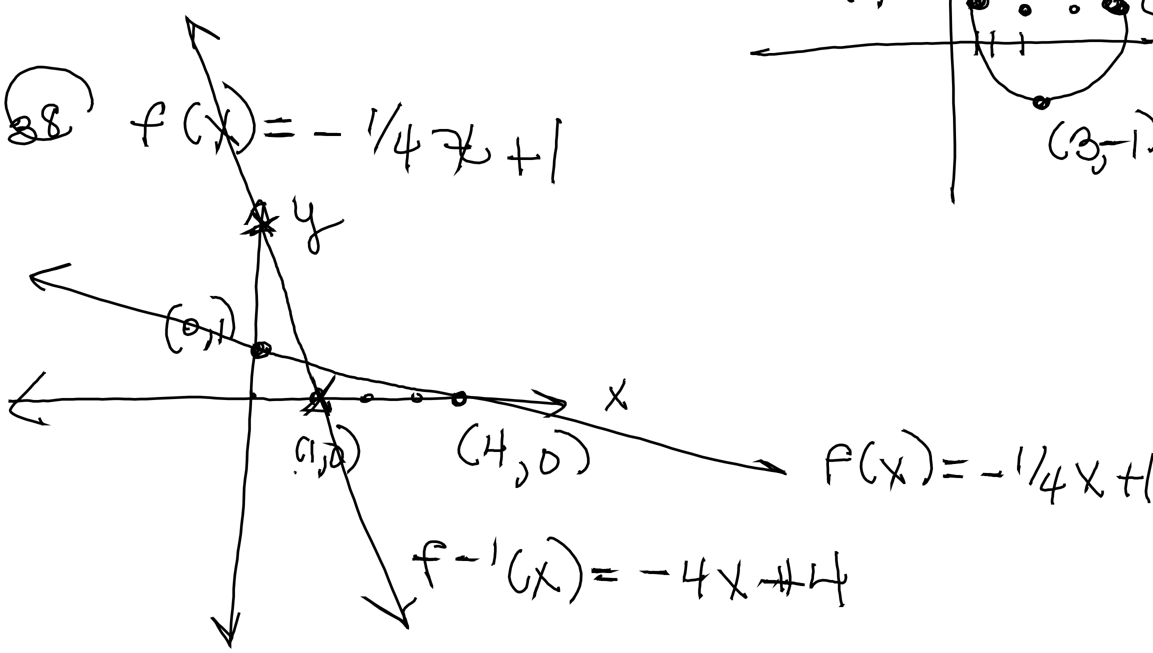
37) $(x-3)^2 + (y-1)^2 = 4$

$(x-h)^2 + (y-k)^2 = r^2$ center (h,k) radius $= r$

center $(3,1)$ $r = 2$



38) $f(x) = -1/4x + 1$



$y = -\frac{1}{4}x + 1 \longrightarrow x = -\frac{1}{4}y + 1$

$-4(x-1) = \left(-\frac{1}{4}\right)y(-4)$

$-4x + 4 = y = f^{-1}(x)$